Sample Mean Sampling Distribution: $\mu_x = \mu_{\bar{x}}$ and $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$

Sample Mean Sampling Dist z-score: $Z = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x}-\mu_x}{\frac{\sigma_x}{\sqrt{n}}}$

Popular Z values:

Confidence	Error Probability	Z
.9	.1	1.65
.95	.05	1.96
.99	.01	2.58

Population Mean Confidence Interval known σ_x: $\mu_{\bar{x}} \pm z\left(\frac{\sigma_x}{\sqrt{n}}\right)$

Population Mean Confidence Interval unknown σ_x: $\mu_{\bar{x}} \pm t_{\frac{\alpha}{2},n-1}\left(\frac{\sigma_x}{\sqrt{n}}\right)$

Test Statistic for Mean Hypothesis Test with unknown σ_x: $t^* = \frac{\bar{x}-\mu_x}{\frac{s_x}{\sqrt{n}}}$

Test Statistic for Mean Hypothesis Test with known σ_x: $z^* = \frac{\bar{x}-\mu_x}{\frac{\sigma_x}{\sqrt{n}}}$

Hypothesis Test Decisions:

Alternative Hypothesis	Probability	P-Value
$H_a: p > p_0$	Right Tail	P(Z>z*)
$H_a: p < p_0$	Left Tail	P(Z<z*)
$H_a: p \neq p_0$	Two Tail	2*P(Z<-|z*|)

1) Suppose a simple random sample of 25 bottles of beer from a population with $\sigma_x = .15$ resulted in a sample mean pour of 15.8 oz.

a. Calculate a 90 confidence interval for the population mean pour.

- We have to assume normality b/c $n = 25 < 30$
- Data is random

$$\bar{x} \pm z\frac{\sigma_x}{\sqrt{n}} = 15.8 \pm (1.65)\frac{.15}{\sqrt{25}} = 15.8 \pm .0495 = (15.7505, 15.8495)$$

We are 90% confident that the true population mean pour is between 15.7505 and 15.8495 ounces. They are under-filling their pints!

b. Test, with 95% confidence that the true population mean pour is 16 oz. Interpret your results. [redacted]

i. State Hypothesis: $H_0: \mu = 16$

$H_a: \mu \neq 16$

ii. Check Assumptions:
- $n = 25 < 30$ so we have to assume normality
- Data is obtained randomly

iii. Calculate Test Statistic

$$z^* = \frac{16 - 15.8}{\frac{.15}{\sqrt{25}}} = \frac{.2}{.03} = 6.\overline{66}$$

iv. Find p-value $2P(Z < -|6.\overline{66}|) = 2P(Z < -6.\overline{66})$

$\approx 2(0)$

$= 0$

v. Interpret With a p-value of 0 we have to reject the null hypothesis in favor of the alternative because $\underset{\text{p-value}}{0} < 1 - .95 = \underset{\alpha}{.05}$. There is enough evidence to suggest That the true population mean pour is not 16 oz.

2) A sample of thirty students' spending in Five Points this semester, var 1, provided the data below at the 95% confidence level. Complete parts a and b.

Variable	Sample Mean	Std. Err.	DF	T-Stat	P-value
var1	209.5	3.9562986	29	2.4012343	0.9885

a. Find a 95% confidence interval for the true population mean of the students' spending in Five Points this semester. **Use $t_{\frac{\alpha}{2},n-1}$=2.045.**

·$n=30$

·Assume a random sample

$$\bar{X} \pm t \frac{S_x}{\sqrt{n}} = 209.5 \pm 2.045(3.9562986) = (201.4094, 217.5906)$$

We are 95% confident that the true population mean spending is between \$201.41 and \$217.59 per student in Five-Points.

b. Test, with 95% confidence that the true population mean of the students' spending in Five Points this semester is less than \$200, with the data above. Interpret your results. (Just fill out the three steps below.)

Hypothesis: $H_0: \mu \geq 200$

$H_a: \mu < 200$

P-value: .9885

Decision: We fail to reject the null hypothesis because $\underset{\text{p-value}}{.9885} > 1 - .95 = \underset{\alpha}{.05}$. This indicates that there is not enough evidence to suggest the mean spending is less than \$200.00.

3) Suppose that among a random sample of 26 classes at University of South Carolina, the sample mean class size is 117.15 students with a sample standard deviation of 93.33.

Variable	Sample Mean	Std. Err.	DF	T-Stat	P-value
Class Size	117.15385	18.303652	25	-1.7945137	0.0424

a. Calculate a 95% confidence interval for the population mean of class size. $t_{\frac{\alpha}{2},n-1}$**=2.06.**

- $n = 26$ → Need to assume normality
- random sample

$$\bar{x} \pm t\frac{s_x}{\sqrt{n}} = 117.15385 \pm 2.06(18.303652) = 117.15385 \pm 37.7055$$
$$= (79.44835,\ 154.85935)$$

We are 95% confident that the true population mean class size is between 79 and 155 students.

b. Test, with 95% confidence that the true population mean class size at University of South Carolina is less than 150, with the data above. Interpret your results. (Just fill out the three steps below.)

Hypothesis: $H_0: \mu \geq 150$

$H_a: \mu < 150$

P-value: .0424

Decision: We reject the null hypothesis in favor of the alternative because $\underset{\uparrow \atop \text{p value}}{.0424} < 1 - .95 = \underset{\uparrow \atop \alpha}{.05}$. Ther is enough evidence to suggest that the true population mean is less than 150.

4) Suppose that among a random sample of 26 classes at University of South Carolina, the sample mean class size is 117.15 students with a population standard deviation of 95.

a. Calculate a 95% confidence interval for the population mean of class size.

· $n=26 \rightarrow$ Need to assume normality
· random sample

$$\bar{x} \pm z \frac{\sigma_x}{\sqrt{n}} = 117.15 \pm 1.96\left(\frac{95}{\sqrt{26}}\right) = 117.15 \pm 36.5168 = (80.6332, 153.6668)$$

We are 95% confident that the true population mean class size is between 81 and 154 students.

b. Test, with 95% confidence that the true population mean class size at University of South Carolina is less than 150, with the data above. Interpret your results. (Just fill out the three steps below.)

i. State Hypothesis: $H_0: \mu \geq 150$
$H_a: \mu < 150$

ii. Check Assumptions: · $n=26 \rightarrow$ Need to assume normality
· random sample

iii. Calculate Test Statistic $z = \dfrac{\bar{x} - \mu_x}{\sigma_x/\sqrt{n}} = \dfrac{117.15 - 150}{95/\sqrt{26}} = \dfrac{-32.85}{18.6310} = -1.7632$

iv. Find p-value $P(Z < z^*) \approx P(Z < -1.76) = .0392$

v. Interpret We reject the null hypothesis in favor of the alternative because $\underset{\text{p-value}}{.0392} < 1 - .95 = \underset{\alpha}{.05}$. There is enough evidence to suggest that the true population mean is less than 150.

5) The average weekly loss of study hours due to consuming too much alcohol on the weekend is studied on 10 students and the number of hours saved by attending a certain alcohol awareness program. The average number of hours saved was 2.1.

Variable	Sample Mean	Std. Err.	DF	T-Stat	P-value
hours saved	2.1	0.56666667	9	-1.5882353	0.9267

a. Calculate a 90% confidence interval for the population mean number of hours saved.

$t_{\frac{\alpha}{2},n-1}$=1.833.

- $n=10 \rightarrow$ Need to assume normality
- need to assume a random sample

$$\bar{x} \pm t\left(\frac{\sigma_x}{\sqrt{n}}\right) = 2.1 \pm 1.833(.5\overline{6}) = 2.1 \pm 1.0387 = (1.0613, 3.1387)$$

We are 90% confident that the true population mean number of hours saved is between 1.0613 and 3.1387 hours; the program works!

b. Test, with 95% confidence that the true population mean number of hours saved is more than 3, with the data above. Interpret your results. (Just fill out the three steps below.)

Hypothesis: $H_0: \mu \leq 3$

$H_a: \mu > 3$

P-value: .9267

Decision: We fail to reject the null hypothesis because

$\underset{\uparrow \text{p-value}}{.9267} > 1 - .95 = \underset{\uparrow \alpha}{.05}$. There is not sufficient evidence

to suggest the true population mean is bigger than three hours.